Operational modal analysis by thermoelasticity

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ABSTRACT: In this work the theoretical and practical possibility of using a new measurement technique, based on high speed thermographic acquisition systems, for the study of the dynamic response of objects has been demonstrated. This methodology has been named thermoelastic modal stress analysis (TMSA). A simple test case is used to illustrate this new measurement technique: a plastic bending beam. Excitation of the structure has been performed by band-limited noise and response is measured in terms of temperature time history on its surface on the recorded thermal film. Temperature time evolution on the beam surface is related to stress time evolution according to the thermoelastic principle, for each excited resonance frequencies. By developing a special data processing software of the thermal film the possibility to get the operational deflection mode shapes of the structure is illustrated.

1. CLASSICAL THERMOELASTIC STRESS ANALYSIS

The thermoelastic stress analysis is based on the thermoelastic measurement principle. This principle is well known in gases: an adiabatic compression or expansion causes a temperature variation. This effect exists in solid and liquid also but the temperature variations are very small in comparison to the gases. Usually a compression of a solid element causes a temperature increase, while a traction state causes a temperature reduction, as illustrated in Fig 1.

The thermoelastic effect induces a very slight temperature variation: in steel, a load that brings it to the elastic limit, induces a temperature variation of 0.2 degree. Suppose to measure temperature on a steel specimen while it is subjected to a step load (Fig 2). As represented in the graph, at the beginning the temperature follows the time history of the load, than it decrease due to the heat transfer through the environment.
Thomson W. (1853) proposed the following equation to relate the stress changes in terms of sum of changes principal stresses $\Delta \sigma_x$ and $\Delta \sigma_y$ with the change of the temperature.

$$\Delta T = \frac{-\alpha T}{\rho C_p} (\Delta \sigma_x + \Delta \sigma_y) \quad (1)$$

Where:
- $\alpha$ Coefficient of thermal expansion.
- $T$ Absolute temperature.
- $\rho$ Density.
- $C_p$ Specific heat.
- $\Delta \sigma_x, \Delta \sigma_y$ Surface stress amplitudes on two perpendicular directions.

The problem is that temperature difference generated by thermoelastic effect and the other generated by other physical phenomena cause heat exchange that can mask the thermoelastic effect. The heat exchange between material and environment and within the material can be reduced by using a high frequency dynamic load. In this case, the temperature time history follows the stress time history, as shown in Fig 3.
Usually, the typical conditions of fatigue tests are suitable to perform Thermoelastic Stress Analysis (TSA). Therefore, during a cyclical loading of a mechanical component, by measuring the amplitude of temperature fluctuation in time on the surface of a mechanical component, it is possible to obtain a map proportional to the first stress invariant fluctuation in time. Measurement on the surface of a mechanical component can be performed using thermocamera, as proposed for the first time by Belgen (1967). A non contact stress measurement technique on material surfaces can be therefore obtained by measuring temperature fluctuation in time. A typical setup of a TSA measurement system is illustrated in fig 4. To detect the amplitude of temperature fluctuation from the time signal of each pixel of the thermocamera, because of the high level of noise normally existing, a lock-in data processing technique is performed, using as reference the loading signal. Harwood N. et al. (1991) and Boyce B.R. (1999).

![Figure 4: A typical thermoelastic measurement chain](image)

2. OPERATIONAL MODAL ANALYSIS BY TERMAL EMISSION

A first attempt to measure mode shapes of blades in terms of stress distribution by thermoelasticity has been performed at Rolls Roice Turner S. R. and Pollard N.G. (1987), but using only sinusoidal excitation and detecting amplitudes of temperature fluctuation by an old scanning thermoelastic measurement system, i.e. not recording thermal films. Another work of this kind has been performed by Beattie A.G. and Rumsey M.A. (1998) on a composite blade, also in this case using sine excitation at resonance frequencies previously determined, using in this case a full frame measuring thermocamera.

The new idea here proposed is to measure a thermal film using an high speed and high resolution thermocamera. Here a Deltatherm 1560 system manufactured by Stressphotonics was used. On each dynamic random time signal recorded on each pixel of the thermal film the estimation of power spectrum is performed. The map of the amplitude of each spectrum peak detected represent the map of the first stress invariant at that frequency, i.e. an operational mode shapes in terms of stress, accordingly to Eq (1).

This measurement techniques could have obvious advantages respect intrusive techniques, using contact sensors like accelerometers for example, but also important advantages respect other non contact optical techniques. The methodology here proposed is full field and with simultaneous data acquisition on each measurement point. Respect for example to holographic techniques, it have the advantage of taking, on a single acquisition, potentially information about many mode shapes, because the thermal film contain the time histories on each pixel (measurement point) so its spectrum contain information about different modes in a complete bandwidth. Respect for example to laser scanning vibrometry it have the advantage of the simultaneous acquisition on each one of the measurement points; a reference phasing signal for the subsequent data acquisition performed by scanning with a single laser beam is not necessary. It is therefore possible analyzing also transient dynamic phenomena.

3. A CLASSICAL SIMPLE REFERENCE TEST CASE

In order to demonstrate the feasibility of this measurement technique, as test case the classical bending beam illustrated in Fig 5 has been chosen to perform our first experiments, Moretti M. (2006).

Figure 5: The first test case

Very well known solution for its mode shapes are available. Its deformation in vertical direction and in time for each n-th mode can be described by a function of the form:

\[ w(x, t) = W_n(x) \cdot T_n(t) \]  

therefore the stress on this beam can be due to bending, an the bending moment on the section at distance x and at time t will be, as well known:

\[ M(\bar{x}, \bar{t}) = -EJ_y \left[ \frac{d^2 w(x, t)}{dx^2} \right]_{x=\bar{x}} \]  

And is also well known that the maximum stress are on the two over and bottom beam surfaces, one in tension and the other in compression. This maximum stress can be calculated by:

\[ \sigma_{xs}(\bar{x}, \bar{t})_{max} = \pm \frac{M(\bar{x}, \bar{t})}{J_y} \frac{s}{2} \]  

where s is the thickness of the beam. When n modes are excited, each one with a weighting factor \( C_i \) the total stress field will be a linear superimposition of the stress due to each i-th mode shape as:

\[ \sigma_{xs}(x, t)_{tot} = \sum_{i=1}^{n} C_i \sigma_{xs}(x, t)_{oi} \]  

For a polyethylene beam, of rectangular section (15 x 3 mm), of length 31 mm with an extreme free and an extreme bonded to an electrodynamics shaker the first four resonance frequencies can be calculated at 7,3 Hz, 45,6 Hz, 127,8 Hz and 250,3 Hz.. From the very well known mode shapes of this simple structure the distribution of the superficial stress on this test beam can be easily calculated by the above illustrated equations.
In Fig 6 the mode shapes in terms of the first invariant of stress field on the beam surface are illustrated.

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In Fig 6 blue and red colours mean compression and traction stress state respectively. The beam has therefore been realized and mounted on a little dynamic exciter as illustrated in Fig 7. In order to excite the first two modes a band-noise in the range 4-100 Hz has been used.

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**Figure 6:** Mode shapes of the test case in terms of first stress invariant theoretical distribution

(a) and (b) show the mode shapes for compression and traction, respectively, for the first mode. (c) and (d) show the mode shapes for compression and traction, respectively, for the second mode.
The response of the beam has been measured in terms of thermal fluctuation, i.e. a thermal film. One of the frame of the film is illustrated in Fig 8.

Figure 8: The thermal output recorded, proportional to the first stress invariant

Figure 9: Position – frequency – amplitude diagram

Fig. 10: Comparison between experimental and theoretical second mode shape of the beam
4 DATA PROCESSING SOFTWARE OF THE THERMAL FILM

A special purpose software has been developed to analyze the thermal film acquired. The software takes from the film the time history of the signal on each pixel in an area defined by the user, the beam surface in this case. The time history correspondent to every pixel (point of the beam) has used to calculate the Power Spectrum Density (PSD). On each PSD the peak values are identified as illustrated in Fig. 8. A map of the spatial distribution of PSD peak value at each resonance frequency is generated. According to the theoretical results, in the range 5 - 100 Hz, two frequencies of resonance are dectected at 7,2 Hz and 46,2 Hz (Fig. 7). The fig. 8 shows the distribution of the peak values obtained at the first resonance frequency. This result is sufficiently compatible with that obtained analytically, at the frequency of 7,4 Hz (Fig. 5). But the mode shapes is affected by significative noise because of the effects of the large motion of the beam at this low frequency, which affect the thermal film quality. In order to reduce the problems of the quality of the film in the thermographic recordings, at first natural frequency, the displacements have been limited used a lower excitation force. The second resonance frequency measured is at 46,2 Hz (the theoretical one is 45,6 Hz). The Fig 9 represents the spatial distribution of the module of the peak values. The difference between the natural theoretical frequency and the measured one, are surely due to the constraint that doesn't correspond perfectly to that of the numerical model. The Figs 10 and 11 show the peak values distributions corresponding to third and fourth natural frequency. In this case the results obtained show a very good compatibility with the expected theoretical distribution.

5 OPERATIONAL MODE SHAPES OF FAN BLADES IN TERMS OF STRESS

Other tests has been performed on fan blades, excited in resonance conditions, in order to better understand and illustrate differences of mode shapes distribution in terms of “mobility” or in terms of stress. Some vibrating fan was installed on an LDS electrodynamic exciter (an LDS shaker, range 4 Hz - 5 Khz, 1500 N) as illustrated in Fig 11.

Figure 11 : The fan installed over the electrodynamic exciter and its first two modes at 63 and 99 Hz

By measuring the frequency response function using a little accelerometer on some point on the blade tip the firs two resonance frequencies at 63 Hz and 99 Hz has been identified. Previous tests on this fan Di Renzo A. et al.(2006) was performed also measuring all the detected modes by a laser scanning vibrometer. The first two are here illustrated in Fig 11.

This blade has a plastic cylinder with the possibility to be fixed inside the metallic central ring realized as two shell. The cylinder is connected to the foil of the blade trough a conic element. A sine excitation at these two frequencies has been performed and in these vibrating condition the two maps of the amplitude of thermal fluctuation by using a thermoelastic measurement system has been measured.

The results obtained are illustrated in Fig 12 a and b.
The two stress distribution shows a maximum concentrated at the cone tip for the first vibrating mode and at the cone side for the second mode.

By using a strain gauge on an area of the blade the amplitude of the temperature fluctuation can be scaled in stress units, Di Renzo A. (2008). A typical map expressed in MPa near the cone tip is illustrated in Fig 12 c.

The same two modes of the same blade has been calculated using a FEM model, Marsili R. et al. (2004)], results obtained for the first two modes in terms of displacement and stress are illustrated in Fig 13. Maximum stress of up to 15 Mpa has been detected on the cone tip.

The most stressed point is the same calculated by FEM model and measured by thermoelasticity technique. The maximum value calculated was 16,8 MPa, the thermoelastic measurement system gives a maximum of 15,0 MPa.

On the same fan was also performed a typical fatigue test. The component was put in vibration on the shaker using a sine excitation at the first resonance frequency with an amplitude of
12 g. After about 36 hours, corresponding to about 8,000,000 of cycle on all the blades of the fan a crack failure starts on the cone tip. A picture of the crack at its beginning is illustrated in Fig. 14.

A second kind of fan was tested. The resonance frequencies has been measured once again by classical methods, using FRF from response measured by an accelerometer and exciting random noise. The tests has been performed by measuring differential thermal maps, proportional to the first stress invariant distribution, using sine excitation at the first resonance frequencies, that for a blade is typically a simple bending mode.

The second fan tested is illustrated in fig 15. Is a plastic injection molded fan, with the blades directly connected to the plastic central ring, linked to a metal flat disk. The first four resonance frequencies of the blades of this fan was fonded at 112, 157, 222 and 298 Hz.
Thermal maps was recorded while the fan was vibrating at 112 Hz on the exciter. A typical results obtained is illustrated in Fig 16. It is very clear the strong stress concentration near the blade constrain at its base similar to the distribution of the simple bending beam previously illustrated. A fatigue test of this kind of blade gives no failure after 12 M of cycle at 10 g at first resonance frequency equal to 111 Hz.

The last blade tested, have a different connection to the central ring of the fan. The plastic foil of the blade is connected to the metallic central disk by four holes on the foil, two metallic plates and four rivets.

![Figure 17: Stress distribution on a blade vibrating in resonance condition and first resonance frequency](image)

Also in this case the same procedure has been performed in order to determine the first resonance frequency by classical technique. The complete fan was therefore mounted on the electrodynamic exciter and forced to vibrate in resonance condition by a sine excitation at the first resonance frequency of the blade, 87.5 Hz in this case. The result obtained, illustrated in Fig 17, clearly show a complex stress distribution and highlight the most stressed parts of the blade foil that are not at its root but near one of the hole.

Also in this case is very clear the possibility given by the operational mode shape measurement in terms of stress by thermoelasticity, to easily identify the most critical parts of this not so simple plastic and metal structure.

For this last example it is not so easy to predict this result by using a fan model because of the complexity of the modeling the connection between the metallic and the plastic part of this structure.

Also this kind of fan gives no failure after more than 12 M cycles at 87,5 Hz with 4 g of amplitude of vibration.

6 CONCLUSION

In this work we have demonstrated the possibility to measure operational modes of mechanical structures by thermoelasticity by data processing of thermal films recorded using a high performance thermographic system.

In this way it is possible put together information on the stress state of a structure with the information related to its dynamic behaviour.

The proposed method presents the limitations in the measured frequencies (we have actually tested up to 150 Hz) due to the limits of the thermographic FPA actually available.

Testing on complex structures of plastic and metallic fans allowed to illustrate clearly the possibility of this measurement technique to identify the most stressed parts of this kind of mechanical components, relative to each operational mode expressed in terms of stress distribution.
7 REFERENCES

Brincker, R., Zhang, L. and Andersen, P. 2000 Output-only Modal Analysis by Frequency Domain Decomposition. ISMA 25.