Development of a new measure methodology for the dynamic behavior study of objects through Thermoelastic Stress Analysis

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ABSTRACT
In this paper measurement of vibrations on a test case, a simple typical bending beam on an electro dynamical exciter, are performed by using a high resolution and sufficiently fast thermocamera. Excitation of the beam has been performed by band-limited noise and response is measured in terms of temperature time history on its surface by the recorded thermal film. Temperature time evolution on the beam surface is also related to stress time evolution according to the thermoelastic principle. By developing a special data processing software of the thermal film the possibility to get the operational deflection mode shapes of the beam is clearly demonstrated. The possibility to measure a new proposed transfer function named stressance (ratio between exciting force and resulting stress) on the different point of the beam surface is proposed.

1. INTRODUCTION
In this work the theoretical and practical possibility of using a new measurement technique, based on high speed thermographic acquisition systems, for the study of the dynamic response of objects has been proposed. The thermographic maps measured on objects, loaded by periodical forces, is processed by mathematical algorithms to have information on the vibration mode shape, in terms of stress distributions. This methodology, as the TSA (Thermoelastic Stress Analysis), has been called TMSA (Thermoelastic Modal Stress Analysis). Surely this method will become very important in the field of the structural engineering, for the possibility to associate the correspondent state of superficial stress to the resonance frequencies of a system. In this way we could have a complete knowledge of the stress condition of a mechanic component. The measurement techniques, here proposed, has obvious advantages respect intrusive techniques, using contact sensors like accelerometers for example, but also important advantages respect other non contact optical techniques. The methodology here proposed is full field and with simultaneous data acquisition on each measurement point. Respect for example to holographic techniques, it have the advantage of taking, on a single acquisition, information about many mode shapes, because the thermal film contain the time histories on each pixel (measurement point) so its spectrum contain information about modes in a complete bandwidth. Respect for example to laser scanning vibrometry it have the advantage of the simultaneous acquisition on each one of the measurement points; a reference phasing signal for the subsequent data acquisition performed by scanning with a single laser beam is not necessary. It is therefore possible analyzing transient dynamic phenomena.

2. THE MEASUREMENT METHODOLOGY
The dynamic characteristics of a mechanical system can be described in terms of resonance frequencies that depends on the degrees of freedom chosen to describe the real system.
The instantaneous deformation of a point P, of a vibration system, can be seen as the overlap of the deflections associated to the single mode shape. [6] [7]
\[ \varepsilon_{i,j}(P,t) = \sum_{k=1}^{K} A_k \cdot \varepsilon_{i,j,k}(P,t) \quad i,j=1,2,3, \]  
where \( K \) is the number of the vibration mode, the coefficients \( A_k \) don't depend on the specific point \( P \), but only on the vibration mode.
The deformation of a system depends on the state of stress that, to a certain instant, is the overlap of the stress \( \sigma \) caused by the individual modes.

\[
\sigma_{i,j}(P,t) = \sum_{k=1}^{n} A_k^* \sigma_{i,j,k}(P,t) \quad i,j=1,2,3; (2)
\]

The \( A_k \) coefficients are the same for the deformation and the stress. \( \sigma_{i,j,k} \) is a sinusoidal function, having the frequency correspondent to the associated mode.

\[
\sigma_{i,j,k}(P,t) = |\sigma_{i,j,k}(P)| \sin(\omega_k t + \phi_k) \quad i,j=1,2,3; (3)
\]

The tensor of the stress, in any \( P \) point of a vibrating structure, has a certain number of fundamental frequencies:

\[
\sigma_{i,j}(P,t) = \sum_{k=1}^{n} A_k^* |\sigma_{i,j,k}(P,t)| \sin(\omega_k t + \phi_k) \quad i,j=1,2,3; (4)
\]

The thermoelastic stress analysis has been able to study the state of superficial stress, as the first stress invariant, through the analysis of the thermal signal originated from the object surface. The above equation, written as first stress invariant, is:

\[
\sigma_{i}(P,t) = \sum_{k=1}^{n} A_k^* |\sigma_{i,k}(P,t)| \sin(\omega_k t + \phi_k) \quad i,j=1,2,3; (5)
\]

The Kelvin law related the first stress invariant to the temperature [4]:

\[
\delta T = \frac{-\alpha T \delta \sigma}{\rho C_p} \quad (6)
\]

where \( \delta \sigma \) is the sum of the stress in two perpendicular direction on the specimen surface (i.e. the first stress invariant); \( \alpha \) is the thermal expansion coefficient; \( T \) is the absolute temperature of the component; \( \rho \) is the density and \( C_p \) is the specific heat at constant pressure.

Combining the two equations (5) and (6) and considering two time instants \( t \) and \( t+dt \) :

\[
\delta T(P,t+dt,t) = -\frac{\alpha T}{\rho C_p} \sum_{k=1}^{n} A_k^* |\sigma_{i,k}(P,t)| \sin(\omega_k (t + dt) + \phi_k) - \sin(\omega_k t + \phi_k) \quad (7)
\]

The temperature time changes can be considered as a linear combination of the stress time changes generated by each individual vibration mode. That is to say in the thermal signal, measured in time, the contribution of the different vibration mode exists together and each of them is characterized by its natural frequency. As each of these contributions has a definite frequency, these contributions could be analyzed and measured. As for the functions of compliance, mobility and inheritance, a new function called stressance is here proposed defined as the complex ratio of the first invariant of strain and force spectrum:

\[
St(\omega) = \frac{\sigma_i(\omega)}{F(\omega)} \quad [N/mm^2 / N] \quad (8)
\]

An example explains this function: the \( T(t) \) signal, illustrated in the following Fig. 1, represents the temperature time history in a point of a structure, proportional to the output signal of the pixel of the thermocamera.

\[
\text{Fig. 1: Temperature time history in a point of a structure detect by the output signal of a pixel of the thermocamera}
\]
In the frequency domain the time history can be observed as the spectrum of figure 2:

![Spectral density of signal of Fig. 1](image)

**Fig. 2: Spectral density of signal of Fig. 1**

The signal is composed by different harmonics that represent the resonance frequencies of the mechanical system: in fact in correspondence of the resonance frequencies the system presents a maximum local of compliance, to which a maximum local of the function $\varepsilon_{ij}(P, \omega)$ corresponds. The Young module and the Piosson’s coefficient relate the deformation to the stress; a stress maximum corresponds to a deformation maximum, in an elastic linear loading conditions. If tangential stress $\tau_{ij}$ don’t exist, then also the $\sigma_{i}(P, \omega)$ function presents a local maximum in correspondence to the resonance. The natural frequencies, not purely torsional, can be analyzed from the thermal signal generated by vibrating structure. Like in the classic modal analysis, applying this procedure on many points, it is possible to determine the superficial map of the first stress invariant, in correspondence to the resonance frequencies. The measurement of the thermal signal on the object surface could be realized with contact methods like thermocouples, or thermo resistance, or with no contact-methods measuring the infrared radiation emitted by a body.

Among the radiation methods the thermographic system consents the measure of the superficial temperature on many points simultaneously: the DeltaTherm System, used in this work, employs a focal plane array (FPA) of 320 x 256 pixel with more than 80000 sensible elements. Therefore this system allows a simultaneous acquisition of time history in about 80000 points on the surface of the mechanical component under study.

### 3. THEORETICAL MODEL OF A SIMPLE EXPERIMENTAL EXAMINED CASE

The test case selected is a polyethylene beam, of rectangular section (15 x 3 mm), with an extreme free and an extreme bonded to an electrodynamics shaker (Fig. 3 and Fig. 4).

For this case the mode shapes are well known, the resonance frequencies are low and the material thermoelastic constant, equal to $\frac{\alpha}{\rho C_{P}}$, is very high.

The distribution of the superficial stress on this test beam can be easily calculated by analytical methods. The figure 5 show the distribution of the first invariant of stress, calculated in correspondence to the first four vibration modes.

![Scheme of the beam and loading conditions](image)

**Fig. 3: Scheme of the beam and loading conditions**

![The loading and testing set-up](image)

**Fig. 4: The loading and testing set-up**
The resonance frequencies of the beam calculated are 7.3 Hz, 45.6 Hz, 127.8 Hz and 250.3 Hz.

4. THE EXPERIMENTAL MEASUREMENTS
A thermal film is recorded while this test beam was vibrating, excited by the electrodynamics shaker as illustrated in Fig. 4.

The fig. 7 shows a typical thermographic frame of the film recorded of the vibrating test beam. The sample frequencies of the thermal film is 200 fps (frames per second). It's therefore possible to measure stress up to 100 Hz, without aliasing. Only the first two modal shape of the test beam are in this range.

The time history of stress acquired at frequencies higher than 100 Hz are affected by aliasing. In this case an excitation in narrow band is used.

The relation between the “real” frequency and the “aliased” can be simplified as:

\[ f_r = \frac{f_{\text{sup}} - f_i}{2n+1} \quad \text{if} \quad f_i = f_{\text{Nyquist}}*(2n+1), \quad n=1,2,3... \quad (9) \]

\[ f_r = f_i + f_{\text{inf}} \quad \text{if} \quad f_i = f_{\text{Nyquist}}*(2n), \quad n=1,2,3 \quad (10) \]

where:

- \( f_r \) is the real frequency
- \( f_{\text{inf}} \) and \( f_{\text{sup}} \) are the upper and lower limits of the excitation frequency band

Fig. 5: Typical superficial stress in correspondence of the first four resonance frequencies

Fig. 6: Typical thermographic image recorded on the vibrating test beam
5. THE DATA PROCESSING TECHNIQUE

A special purpose software has been developed to analyze the thermal film acquired. The software takes from the film the time history of the signal on each pixel. The time history correspondent to every pixel (point of the beam) has used to calculate the Power Spectrum Density (PSD). On each PSD the peak values are identified as illustrated in Fig. 7. A map of the spatial distribution of PSD peak value at each resonance frequency is generated. According to the theoretical results, in the range 5 - 100 Hz, two frequencies of resonance are calculated at 7.2 Hz and 46.2 Hz (Fig. 7).

The fig. 8 shows the distribution of the peak values obtained at the first resonance frequency. This result is sufficiently compatible with that obtained analytically, at the frequency of 7.4 Hz (Fig. 5). But the mode shapes is affected by significative noise because of the effects of the large motion of the beam at this low frequency, which affect the thermal film quality. In order to reduce the problems of the quality of the film in the thermographic recordings, at first natural frequency, the displacements have been limited used a lower excitation force.

The second resonance frequency measured is at 46.2 Hz (the theoretical one is 45.6 Hz). The figure 9 represents the spatial distribution of the module of the peak values. The difference between the natural theoretical frequency and the measured one, are surely due to the constraint that doesn't correspond perfectly to that of the numerical model.

The figures 10 and 11 show the peak values distributions corresponding to third and fourth natural frequency. In this case the results obtained show a very good compatibility with the expected theoretical distribution.

6. CONCLUSION

In this work we have demonstrated the possibility of measuring the natural frequency and the mode shapes of the vibration mode of mechanical components through the application of thermoelastic data processing techniques on thermal films recorded using an high performance thermographic system. In this way it is possible put together information on the stress state of a structure with the information related to its dynamic behavior, simply by a data processing of thermographic films. The proposed method presents the limitations in the measured frequencies (we have actually tested up to 100 Hz) due to the limits of the thermographic FPA actually available destined to disappear with the technological evolution of the FPA sensor matrix. Anyway the possibility to perform measurements of the only mode shapes at higher frequency, also in aliasing conditions, if is known the resonance frequencies, is demonstrated.
7. REFERENCES


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**Fig. 8:** Typical peak values distribution of stress at first natural frequency

**Fig. 9:** Typical peak values distribution of stress at second natural frequency

**Fig. 10:** Typical peak values distribution of stress at third natural frequency

**Fig. 11:** Typical peak values distribution of stress at fourth natural frequency